Exercise: Amari oscillator

The Amari oscillator is a system of an excitatory neuron, u, coupled to an inhibitory neuron, v as follows:

$$\begin{aligned} \tau \dot{u}(t) &= -u(t) + h_u + w_1\ \sigma(u(t)) - w_2\ \sigma(v(t)) \\ \tau \dot{v}(t) &= -v(t) + h_v + w_3\ \sigma(u(t)) \end{aligned}$$

In words: The excitatory neuron is self-excitatory (w_1) and excites the inhibitory neuron (w_3), while the inihibory neuron inhibits the excitatory neuron (w_2). The sigmoidal function can be approximated as a step-function:

$$\sigma(u) = \begin{cases} 1 & \text{for } u > 0 \\ 0 & \text{for } u < 0 \end{cases}$$

1. Analyze this dynamics exploiting the piece-wise linear nature of the vector-field (in each of the four quadrants, the equation is linear). Determine the fixed point in each quadrant. An oscillation occurs if for each quadrant, the fixed point lies in the neighboring quadrant. Derive parameter conditions such that this is the case (you can use a plotting option at the bottom of `simulator.m` to explore these conditions).

2. Simulate the Amari oscillator. Try to enlarge the limit cycle or make it smaller by varying the parameters.