Exercise: analytical work for piece-wise linear dynamics

The neuronal dynamics simulated in the lectures is

$$\tau \dot{u}_1(t) = -u_1(t) + h_1 + S_1(t) + c_{11}\sigma(u_1(t)) + c_{12}\sigma(u_2(t))$$
$$\tau \dot{u}_2(t) = -u_2(t) + h_2 + S_2(t) + c_{22}\sigma(u_2(t)) + c_{21}\sigma(u_1(t))$$

The sigmoidal function is given by

$$\sigma(u) = \frac{1}{1 + \exp[-\beta\ u]}$$

. For large values of β, the sigmoidal function can be approximated as a step-function:

$$\sigma(u) = \begin{cases} 1 & \text{for } u > 0 \\ 0 & \text{for } u < 0 \end{cases}$$

Analyze this dynamics exploiting the linear approximation of the sigmoidal function: In each of the four quadrants of the state space, (u_1, u_2), the dynamics is linear.

1. Study first the case without coupling ($c_{12} = C_{21} = 0$). Determine the fixed point in each quadrant. In each case, examine in which quadrant that fixed point lies. If it lies in the quadrant for which it was computed, then this quadrant contains a stable fixed point. If it lies outside, then the vector-field in this quadrant only generates transients in the direction of the fixed point. Derive conditions for input strength, S_1 and S_2 for the existence of the fixed point attractors.

2. Now study the coupled system ($c_{12} = C_{21} \neq 0$). Derive conditions for the existence of fixed points in the upper right quadrant when inputs are present and inhibitory coupling increases.