Exercise: Three-layer Dynamic Field model

The three-layer Dynamic Field model consists of an excitatory layer, $u(x,t)$, which receives afferent sensory input, $S(x,t)$, a shared inhibitory layer, $w(x,t)$, and second excitatory layer, $w(x,t)$ that receives excitatory input primarily from the first excitatory layer.

$$\begin{aligned}
\tau\dot{u}(x,t) &= -u(x,t) + h_u + S(x,t) + \int dx'\ c_{\rm uu}(x-x')\ \sigma(u(x',t)) \\
&- \int dx'\ c_{\rm uv}(x-x')\ \sigma(v(x',t)) + \int dx'\ c_{\rm uw}(x-x')\ \sigma(w(x',t)) \\
\tau\dot{v}(x,t) &= -v(x,t) + h_v \\
&+ \int dx'\ c_{\rm vu}(x-x')\ \sigma(u(x',t)) + \int dx'\ c_{\rm vw}(x-x')\ \sigma(w(x',t)) \\
\tau\dot{w}(x,t) &= -w(x,t) + h_w + \int dx'\ c_{\rm ww}(x-x')\ \sigma(w(x',t)) \\
&- \int dx'\ c_{\rm wv}(x-x')\ \sigma(v(x',t)) + \int dx'\ c_{\rm wu}(x-x')\ \sigma(u(x',t))
\end{aligned}$$

The kernels, $c_{ij}(x-x')$, projecting across the levels ($i \neq j$) or mediating lateral excitatory interaction ($i = j$) are all gaussian with positive strength, but may vary in width and strength:

$$c_{ij}(x-x') = c_{i,j,\rm strength}\ \exp\left[-\frac{(x-x')^2}{2\sigma_{ij}^2}\right].$$

External input localized around $x_{\rm input}$ is supplied only to the u-layer in the form

$$S(x,t) = S_{\rm strength} \exp\left[-\frac{(x-x_{\rm input})^2}{2\sigma_{\rm input}^2}\right].$$

The sigmoidal function is given by

$$\sigma(u) = \frac{1}{1+\exp[-\beta\ u]}.$$

Use the interactive simulator `interactive_sim3l` to understand how this model works.

1. Set the parameters of the u- and w- layer identically. Induce a peak in the u-layer through input and then turn input off. At appropriate setting, you should see emergent working memory in that a peak is induced in both the u- and the w- layer by the input, but only the w-layer peak is sustained when input is removed.

2. Try to reproduce change detection: induce a peak through input and remove input, leaving a sustained peak in the w-layer. Then try to induce a peak with the same input strength at the same versus at a different location.

3. Have fun trying more things...