An Ambition-Theoretic Approach to Legislative Institutional Choice

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Technical appendix

Proof of Proposition 1:

To begin, because the benefits of office-holding are constant from the point of view of the candidates in the campaign, I collapse the benefit terms so that $A = \alpha b_p + (1-\alpha) b_D$ and $\Delta = \delta b_p + (1-\delta) b_D$. This reorganizes the utility functions into the simpler

$$U_1 = \left(\frac{c_1}{c_1 + c_2}\right)\left(\frac{1}{k^{b_p + b_D}}\right) A - c_1 \tag{1}$$

$$U_2 = \left\{\frac{c_2}{c_1 + c_2} + \left(1 - \left(\frac{1}{k^{b_p + b_D}}\right)\left(\frac{c_1}{c_1 + c_2}\right)\right)\right\} \Delta - c_2 \tag{2}$$

In a Nash equilibrium in the election game, each candidate's cost and effort must be a best response to the other player's effort or incurred campaign cost (as well as to any other salient aspects of the environment). The first-order conditions that express this are

$$\frac{\partial U_1}{\partial c_1} = \frac{A}{(c_1 + c_2) k^{b_p + b_D}} - \frac{c_1 A}{(c_1 + c_2)^2 k^{b_p + b_D}} - 1 = 0 \tag{3}$$

$$c_1^* = \sqrt{\frac{Ac_2}{k^{b_p+b_D}}} - c_2 \tag{4}$$

and

$$\frac{\partial U_2}{\partial c_2} = \left\{ \frac{1}{c_1+c_2} - \frac{c_2}{c_1+c_2} - \left(1 - \frac{1}{k^{b_p+b_D}}\right)\left(\frac{c_1}{(c_1+c_2)^2}\right) \right\} \Delta - 1 = 0 \tag{5}$$

$$c_2^* = \sqrt{\frac{\Delta c_1}{k^{b_p+b_D}}} - c_1 \tag{6}$$

Additionally, equations 4 and 6 will satisfy the second-order requirements for maxima – at the appropriate values, $\frac{\partial^2 u_1}{\partial c_1^2} = -\frac{2}{\sqrt{\frac{Ac_2}{k^{b_p+b_D}}}}$ and $\frac{\partial^2 u_2}{\partial c_2^2} = -\frac{2}{\sqrt{\frac{\Delta c_1}{k^{b_p+b_D}}}}$. These expressions will be negative (ensuring that they find maxima) when the variables and parameters underneath the square root signs are positive. A, Δ, and $k^{b_p+b_D}$ are all constrained by assumption to be positive, so whenever the other player expends any effort, these are the utility-maximizing hypersurfaces or reaction functions for each player.

Proof of Proposition 2:

In equilibrium, $c_2 = c_2^*$, which implies that

$$c_1 = \sqrt{\frac{A\left(\sqrt{\frac{\Delta c_1}{k^{b_p+b_D}}} - c_1\right)}{k^{b_p+b_D}}} - \sqrt{\frac{\Delta c_1}{k^{b_p+b_D}}} + c_1 \tag{7}$$

Simplifying this expression leads to

$$\sqrt{\frac{A}{k^{b_p+b_D}}\sqrt{\frac{\Delta c_1}{k^{b_p+b_D}}}} = \sqrt{\frac{\Delta c1}{k^{b_p+b_D}}} \tag{8}$$

And solving for c_1 yields

$$c_1^* = \frac{A^2\Delta}{k^{b_p+b_D}(A+\Delta)^2} \tag{9}$$

Substituting Equation 9 back into Equation 6 yields

$$c_2^* = \sqrt{\frac{\Delta \frac{A^2\Delta}{(A+\Delta)^2 k^{b_p+b_D}}}{k^{b_p+b_D}}} - \frac{A^2\Delta}{(A+\Delta)^2 k^{b_p+b_D}} \tag{10}$$

Which implies that

$$c_2^* = \frac{A\Delta^2}{(A+\Delta)^2 k^{b_p+b_D}} \tag{11}$$

Which completes the proof.

Proof of Proposition 3

The equilibrium values for c_1^* and c_2^* can now be substituted back into the legislator's utility function to determine what shape a sitting legislator's preferences for the institutions of the next session takes. After substituting and simplifying,

$$U_1 = \frac{A^3}{(A+\Delta)^2 k^{b_p+b_D}} \tag{12}$$

And re-expanding the collapsed terms yields

$$U_1 = \frac{(\alpha b_p + (1-\alpha)b_D)^3}{((\alpha b_p + (1-\alpha)b_D) + (\delta b_p + (1-\delta)b_D)^2 k^{b_p+b_D}} \tag{13}$$

Which completes the proof.

Proof of Proposition 4

Consider the border solutions where $b_p = 0$ or $b_D = 0$. When $b_D = 0$,

$$U_1 = \frac{(\alpha b_p)^3}{(\alpha b_p + \delta b_p)^2 k^{b_p}} \tag{14}$$

This can be differentiated and solved for the constrained maximum of U_1. After some simplification and rearrangement,

$$\frac{\partial U_1}{\partial b_p} = -\frac{\alpha^3 k^{-b_p}(-1 + b_p \ln(k))}{(\alpha+\delta)^2} \tag{15}$$

$$b_p^* = \frac{1}{\ln(k)} \tag{16}$$

Note that this holds independent of α and δ whenever $b_D = 0$. The second-order condition for a maximum also holds.Simplified, $\frac{\partial^2 U_1}{\partial b_p^2}$, evaluated at $b_p^*, = -\frac{\alpha^3 \ln(k)}{e(\alpha+\delta)^2}$, which is guaranteed to be negative because all of the variables and parameters to the right of the negative sign are themselves positive. Here, it should be noted that the e in the second-order condition is the root of the natural logarithm and not a parameter of the model.

When $b_p = 0$,

$$U_1 = \frac{((1-\alpha)b_D)^2}{((1-\alpha)b_D + (1-\delta)b_D)^2 e^{b_D}} \tag{17}$$

The constrained maximum of U_1 along this line can also be found:

$$\frac{\partial U_1}{\partial b_D} = \frac{(-1+\alpha)^3 k^{-b_D}(-1+b_D\ln(k))}{(-2+\alpha+\delta)^2} \tag{18}$$

$$b_D^* = \frac{1}{\ln(k)} \tag{19}$$

Again, $b_D^* = \frac{1}{\ln(k)}$ irrespective of the values of α and δ when $b_p = 0$. This result also satisfies the second-order conditions. As in the case where $b_D = 0$,$\frac{\partial^2 U_1}{\partial b_D^2}$, evaluated at $b_D^*, = -\frac{\alpha^3 \ln(k)}{e(\alpha+\delta)^2}$.

Proof of Proposition 6 :

To begin, note that the level of provided benefits will, in equilibrium, be $0, \frac{1}{\ln(k)}$ or $\frac{1}{\ln(k)}, 0$ as equations 16 and 19 show. This implies two cases: one where b_p is positive and one where b_D is positive.

Case 1: $b_p = \frac{1}{\ln(k)}, b_D = 0$

When $b_p = \frac{1}{\ln(k)}$ and $b_D = 0$, c_1^* simplifies to

$$c_1^* = \frac{\alpha^2\delta\exp(-1)}{\ln(k)(\alpha+\delta)^2} \tag{20}$$

Differentiating with respect to k leads to

$$\frac{\partial c_1^*}{\partial k} = -\frac{\alpha^2\delta\exp(-1)}{k\ln(k)^2(\alpha+\delta)^2} \tag{21}$$

This will always be negative since all of the terms are positive and are multiplied by -1. $\ln(k)$ is assured to be positive by the assumption that $k > 1$, but even if k were less than 1 (implying an electoral bonus rather than penalty for higher levels of benefits) , the squaring of the resulting number also assures that it will be positive. This means that c_1^* is falling in k, and since lower k implies higher provided b_p, c_1^* and b_p are directly related, which completes the proof.

Case 2: $b_p = 0, b_D = \frac{1}{\ln(k)}$

The proof for this case proceeds as before except that

$$c_1^* = \frac{(1-\alpha)^2(1-\delta)\exp(-1)}{\ln(k)(2-\alpha+\delta)^2} \tag{22}$$

and

$$\frac{\partial c_1^*}{\partial k} = -\frac{(1-\alpha)^2(1-\delta)\exp(-1)}{k\ln(k)^2(2-\alpha-\delta)^2} \tag{23}$$

This equation will always be negative, assuring that c_1^* and b_D are positively related, for the same reasons as in the first case.

Similar proofs can be easily constructed for c_2^* by moving the square in the numerator of these equations from the α term to the δ.