# ANALYZING A PRIORI KNOWLEDGE

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There are four distinct approaches to analyzing the concept of a priori knowledge. The approaches can be distinguished by posing two questions:

1. What is the primary target of the analysis?
2. Does the analysis of the primary target presuppose some general theory of knowledge or justification?[1]

There are two primary targets of analysis. A reductive approach analyzes the concept of a priori knowledge in terms of the concept of a priori justification: S knows a priori that p just in case S's belief that p is justified a priori and the other conditions on knowledge are satisfied. The primary target of analysis is the concept of a priori justification. A nonreductive approach offers an analysis of the concept of a priori knowledge in terms of conditions that do not involve the concept of the a priori. The primary target of analysis is the concept of a priori knowledge. There are two approaches to analyzing the primary target. A theory-neutral approach provides an analysis that does not presuppose any general theory of knowledge or justification. A theory-laden approach provides an analysis that does presuppose some general theory of knowledge or justification (call it *the background theory*).

Those who embrace a theory-laden analysis incur a special burden: they must separate the features of their analysis that are constitutive of the a priori from those that are constitutive of the background theory. The constitutive features of the a priori are those that *differentiate* it from the a posteriori. The constitutive features of the background theory are those that are necessary for knowledge or justification; they are *shared* by a priori and a posteriori knowledge. My goal is to illustrate how the failure to separate these features leads to erroneous conclusions about the nature of a priori knowledge.

Philip Kitcher (1983) offers the following influential argument in support of the conclusion that mathematical knowledge is not a priori: (1) the concept of a priori knowledge entails that a priori warrant is indefeasible by experience; but (2) the warrant conferred by alleged a priori sources of mathematical knowledge is defeasible by experience. Kitcher's critics, including myself, reject (1). Kitcher (2000) provides two divergent responses. The first concedes that his original supporting argument for (1) is flawed, but offers a new multi-faceted defense. The second contends that the important question about mathematical knowledge is not whether it is a priori but whether it is tradition-independent.

I defend three theses in this paper. First, Kitcher's defense of (1) is undercut by a failure to distinguish the requirements of the reliabilist background theory that he originally favored from those constitutive of the a priori. Second, his contention that the important question about mathematical knowledge is whether it is tradition-independent is also undercut by a failure to distinguish the requirements of the socio-historical background theory that he currently favors from the those constitutive of the a priori. Third, once they are distinguished, we can see that the question of whether mathematical knowledge is a priori remains central to the current debate.

# I

Kitcher (2000, 67) maintains that

> *X* knows a priori that *p* iff *X* knows that *p* and *X*'s knowledge that *p* was produced by a process that is an a priori warrant for *p*.

> α is an a priori warrant for *X*'s belief that *p* just in case α is a process such that for any sequence of experiences sufficiently rich for *X* [to acquire the concepts in] *p*
>
> (a) some process of the same type could produce in *X* a belief that *p*;
>
> (b) if a process of the same type were to produce in *X* a belief that *p*, then it would warrant *X* in believing that *p*;
>
> (c) if a process of the same type were to produce in *X* a belief that *p*, then *p*.

The analysis is theory-laden and reductive. Kitcher (2000, 66) maintains that his "general understanding of warrants is a version of reliabilism," and he (1983, 25) identifies "the general notion of warrant" with Goldman's (1979) notion of justification.[2]

Kitcher's critics focus on conditions (b) and (c), which share a common feature: both impose higher standards on a priori justification than those required by his background theory. I (1988, 2003) have argued that, in the absence of some compelling supporting argument, the higher standards are *ad hoc* and should be rejected.

The problem is clear in the case of condition (c), which precludes the possibility of a priori justified false beliefs. Reliabilism does not preclude the possibility of empirically justified false beliefs. So what is the basis for the higher standard on a priori justification? Kitcher (2000, 72) maintains that (c) is a consequence of (b). Thus, the burden of supporting the higher standards falls entirely on supporting (b); and, moreover, supporting it independently of (c). Condition (b) requires that S's a priori justified belief that p be indefeasible by experience in any world in which S has sufficient experience to acquire the concepts in p. Reliabilism, however, does not preclude the possibility of empirically defeasible justified beliefs. So what is the basis for the higher standard on a priori justification?

Kitcher (1983, 89) originally argued that the higher standard is supported by the intuitive idea that a priori knowledge is independent of experience:

> But if alternative experiences could undermine one's knowledge then there are features of one's current experience which are relevant to the knowledge, namely those features whose *absence* would change the current experience into the subversive experience. The idea of the support lent by kindly experience is the obverse of the idea of the defeat brought by uncooperative experience.

In response, I (1988, 220-221) questioned his account of the relationship between supporting and defeating evidence. It is uncontroversial that if S's belief that p is supported (i.e., justified) by experience then S's belief that p is not justified (and, hence, not known) a priori. But suppose that S's belief that p is justified nonexperientially and that S's nonexperiential justification for the belief that p is defeasible by experience. From the fact that S's justification for the belief that p is defeasible by evidence of kind K, it does not follow that S's belief that p is supported (i.e., justified) by evidence of kind K.[3] Kitcher now concedes this point and agrees that his original defense of (b) fails.

II

Kitcher's new defense consists of a series of arguments that purport to reveal the shortcomings of the rival conception of the a priori favored by his critics. I follow Kitcher in referring to his conception, which includes both conditions (a) and (b), as the *Strong conception* or (SC); and the rival conception, which includes only (a), as the *Weak conception* or (WC). Let us introduce the term 'nonexperiential process' to refer to processes that are available independently of experience. We can now articulate the two conceptions as follows:

(SC) S's belief that p is justified a priori iff S's belief that p is justified by a nonexperiential process and that justification cannot be defeated by experience.

(WC) S's belief that p is justified a priori iff S's belief that p is justified by a nonexperiential process.

There is a further complication that must be addressed before turning to Kitcher's arguments against (WC).

Kitcher no longer endorses the background theory of knowledge that informed his original defense of (SC). He now rejects reliabilism in favor of a socio-historical conception of knowledge. As a result, his arguments against (WC) fall into two categories: (a) those that presuppose reliabilism; and (b) those that presuppose socio-historicism. I address the former in this section and the latter in the next section.

Kitcher offers three arguments against (WC) that presuppose reliabilism. The first alleges that (WC) is satisfiable only if (SC) is satisfiable. The second contends that (WC) fails to capture an important feature of the traditional conception of the a priori. The third maintains that (WC) is too weak.

In support of the contention that (WC) is satisfiable only if (SC) is satisfiable, Kitcher (2000, 74) invites us to "envisage Gauss or Dedekind or Cantor coming to a priori knowledge that nobody has had before on the basis of some kind of process (call it 'intuition')." Let us assume that intuition is sufficiently reliable to produce justified beliefs but that there are possible experiences that call into question the reliability of intuition. Here it appears that a proponent of (WC) is in a position to argue that since these undermining experiences are not present in the actual situation, mathematical beliefs formed on the basis of intuition are justified a priori, whereas a proponent of (SC) must concede that such beliefs are not justified a priori.

Kitcher, however, maintains that this difference is only apparent. In support of this claim, he cites three considerations: (C1) the ability of mathematicians such as Cantor to attain knowledge of new mathematical principles is rare; (C2) since it is rare, it is difficult to find others who can verify that it has been exercised appropriately; and (C3) the history of mathematics indicates that the exercise of this ability has had variable results. Kitcher (2000, 75) concludes that "appeals to elusive processes of a priori reason ought always to be accompanied by doubts about whether one has carried out the process correctly, and whether, in this instance, the deliverances are true." The upshot of this conclusion is that "The power [of intuition] to warrant belief in the actual situation would be undermined—and, indeed, we might claim that one couldn't satisfy the Weak conception unless the Strong conception were also satisfied" (Kitcher 2000, 75).

Kitcher's argument raises two questions. Is it sound? If it is sound, does it provide a basis for preferring (SC) over (WC)? Kitcher contends that Cantor's mathematical beliefs, although reliably produced by the process of intuition, are unjustified. They are unjustified because the justification conferred on them by the process of intuition is undermined by the fact that

(D) Cantor ought always to have doubts about whether he has exercised the process correctly and whether the resulting beliefs are true.

Let us grant that if Cantor's mathematical beliefs are accompanied by such doubts then his justification for those beliefs is undermined. But why should we suppose that his exercise of the process of intuition ought always to be accompanied by such doubts?

Kitcher maintains that (D) is a consequence of (C1)-(C3). But (C1)-(C3) fail to support (D) since it need not be the case either that Cantor believes (C1)-(C3) or that his cognitive state justifies him in believing (C1)-(C3). Moreover, reliabilism does not require that Cantor believes (C1)-(C3) or that his cognitive state justifies him in believing (C1)-(C3). If he does not believe (C1)-(C3) and his cognitive state does not justify him in believing (C1)-(C3), however, then it is hard to see on what basis Kitcher can sustain the claim that Cantor's exercise of the process of intuition ought to be accompanied by such doubt. But if Cantor's exercise of the process of intuition need not be accompanied by such doubts, then it is possible that Cantor's mathematical beliefs satisfy (WC) but not (SC).

But suppose that Kitcher's argument is sound. Does it provide any basis for favoring (SC) over (WC)? No. Kitcher introduced (SC) in order to offer the following argument against mathematical apriorism: (1) the concept of a priori knowledge entails that a priori warrant is indefeasible by experience; but (2) the warrant conferred by alleged a priori sources of mathematical knowledge is defeasible by experience. If (WC) is satisfiable only if (SC) is satisfiable, then (WC) is sufficient to secure the truth of (1). But, if (WC) is sufficient to secure the truth of (1), then there is a strong dialectical reason to abandon (SC). Embracing (SC) requires offering some rationale for imposing higher standards on a priori justification than those required by the background theory of justification. Why incur the burden of supporting (SC) if you can get what you want for free?

Kitcher's (2000, 77) second contention is that (WC) fails to capture a feature of the traditional conception of a priori knowledge: "the tradition ascribes to a priori knowledge the functional significance of being in a position to prescribe to future experience; knowledge that prescribes to future experience is irrefutable by future experience." Let us grant that the tradition ascribes to a priori knowledge the functional significance of prescribing to future experience. This observation, taken by itself, does not provide a basis for favoring (SC) over (WC). The reason is straightforward: Kitcher has not addressed whether this feature is constitutive of the traditional concept of knowledge. If it is constitutive of the traditional concept of knowledge, then it is not a feature that differentiates a priori knowledge from empirical knowledge and, hence, it is not constitutive of the a priori.

It is generally accepted that the traditional concept is Cartesian foundationalism:

> Descartes, along with many other seventeenth- and eighteenth-century philosophers, took it that any knowledge worthy of the name would be based on cognitions the truth of which is guaranteed (infallible), that were maximally stable, immune to ever being shown to be mistaken (incorrigible), and concerning which no reasonable doubt could be raised (indubitable). (Alston 1992, 146)

If this characterization is correct, then it follows that, according to the tradition, incorrigibility is a necessary feature of knowledge and, *a fortiori*, a necessary feature of a priori knowledge. It is not a feature that is constitutive of the a priori. Therefore, the fact that the tradition ascribes to a priori knowledge the functional significance of prescribing to future experience, taken by itself, provides no basis for preferring (SC) over (WC).

Moreover, once we clearly distinguish the requirements of the traditional Cartesian concept of knowledge from the requirements of the a priori, we are in a position to see that (WC) is actually more consonant with the tradition than (SC). The reason is again straightforward. Given the Cartesian concept of knowledge, the distinctive condition of (SC) is both redundant and misleading. It is redundant since the Cartesian concept of knowledge guarantees that it is satisfied. It is misleading since it suggests that irrefutability by future experience is a feature that differentiates a priori knowledge from empirical knowledge.

Kitcher's third contention is that (WC) is too weak. Consider the following thought experiment. Suppose that a cubical die, which is made of some homogeneous material and whose faces are numbered 1 through 6, is rolled once. What is the chance that the uppermost face will be the one numbered 6? One might reason as follows: The material is homogeneous. Therefore, the situation is symmetrical with respect to the six faces. One of the numbered faces will be uppermost. Therefore, the probability that it will be the one numbered 6 is 1/6. Kitcher (2000, 78) maintains that the process involved in this thought experiment is nonexperiential and meets reliabilist standards. Therefore, (WC) has the consequence that the conclusion in question is known a priori.

Kitcher's contention that the conclusion in question meets reliabilist standards is tenuous. But suppose that it should turn out that the belief forming process in question is reliable. Why is this result problematic for (WC)? Kitcher (2000, 79) contends that "this will set the Weak conception at variance with the classical view of the bounds of apriority." This contention is mistaken because it fails to distinguish the requirements of (WC) from those of the background theory in which it is embedded. If (WC) is embedded within the classical Cartesian theory of knowledge then it does not deliver results at variance with the classical theory. In order for a belief to be justified or known a priori within a Cartesian theory of knowledge, it must meet the general conditions on justification and knowledge imposed by that theory. Since Kitcher's example does not meet those conditions—it is neither infallible nor incorrigible nor indubitable—it is not justified or known a priori on the classical view. Kitcher generates the appearance of variance between (WC) and the classical theory by embedding (WC) within a reliabilist theory of knowledge. The variance is due entirely to the difference in the background theories of knowledge in which (WC) is embedded. It is not due to (WC).

III

Kitcher's remaining arguments against (WC) derive from a shift in his background theory of knowledge. Kitcher (2000, 80) now rejects reliabilism in favor of socio-historicism:

> On my *socio-historical* conception of knowledge, the knowledge we have today isn't simply a matter of what we have experienced or thought during the course of our lives, but is dependent on the historical tradition in which we stand and on the social institutions that it has bequeathed to us.

Kitcher maintains that socio-historicism has two significant consequences. First, it shows that only (SC) underwrites the classical view that a priori knowledge is tradition-independent. Second, it reveals that the primary issue regarding mathematical knowledge is not whether it is a priori but whether it is tradition-independent.

Kitcher supports the claim that only (SC) underwrites the tradition-independence of a priori knowledge by alleging that Frege provided an a priori route to mathematical knowledge that Frege regarded as tradition-independent. If Frege were employing (WC), however, that route would be tradition-dependent:

> Suppose that the conception of a priori knowledge employed in these discussions were just the Weak conception. Then there are possible lives, given which processes that would normally warrant a belief in various mathematical propositions would fail to do so. Now imagine a historical tradition whose members have such experiences in the generation that precedes ours. There are two possibilities: in socializing us they either respond to the subversive experiences by explicitly identifying certain processes as unreliable . . . or they do not. If they do, then we are not warranted in believing parts of mathematics on the basis of the process, any more than someone who has been told about mirages is warranted by his perceptions in believing that there is an oasis in the distance . . . . If they do not, then we are still not warranted, for our epistemic situation is akin to that of people reared in a community of dedicated clairvoyants who ignore evidence that their chosen methods are unreliable. . . . Hence, . . . our knowledge turns out to be tradition-dependent. (Kitcher 2000, 82)

Kitcher also contends that if (SC) is adopted, this argument is blocked and the tradition-independence of mathematical knowledge is preserved.

Kitcher's argument is not transparent. I offer the following reconstruction. Let us begin with the simplifying assumption that, for Frege, a single process Φ is the source of all mathematical knowledge. Consider now some mathematical proposition that p and assume that it is justified a priori by Φ:

(A1) S's belief that p is justified a priori by Φ.

Kitcher invites us to assume that Frege is employing (WC):

(A2) (WC).

On the basis of this assumption, he concludes in the second sentence of the quoted passage that

(C1) Therefore, the justification conferred on S's belief that p by Φ is defeasible by experience.

Kitcher invites us to imagine that S's socializers have such experiences—i.e., experiences that defeat the justification conferred on their beliefs by Φ. Since those experiences provide evidence that Φ is unreliable, he concludes that

(C2) Therefore, it is possible that S's socializers have experiential evidence that Φ is not reliable.

He then argues by dilemma in support of the following epistemic principle:

(EP) If S's belief that p is produced by a reliable process Φ and S's socializers have evidence that Φ is not reliable, then the justification conferred on S's belief that p by Φ is defeated.[4]

Since the justification of S's belief that p depends on whether or not her socializers have evidence that Φ is not reliable, it follows that

(C3) S's justification for the belief that p is tradition-dependent.

Kitcher's argument is flawed. Its initial step rests on a critical misunderstanding of (WC). Kitcher alleges that it is a consequence of (WC) that if S's justification for the belief that p is a priori, then S's justification is defeasible by experience. (WC), however, does not entail that a priori justification is defeasible by experience. It entails only that defeasibility by experience is compatible with a priori justification. Since the inference from (A1) and (A2) to

(C1) is invalid, Kitcher's argument fails to establish that (WC) leads to the tradition-dependence of a priori knowledge.[5]

Kitcher's contention that his argument is blocked if (SC) is adopted is also mistaken. Consider again some mathematical proposition that p and assume that it is justified a priori by Φ:

(A1) S's belief that p is justified a priori by Φ.

Now assume that Frege is employing (SC):

(A2*) (SC).

The conjunction of (A1) and (A2*) entails

not-(C1) Therefore, the justification conferred on S's belief that p by Φ is not defeasible by experience.

Not-(C1), however, does not entail

not-(C3) S's justification for the belief that p is not tradition-dependent.

Kitcher overlooks the possibility of *nonexperiential* evidence that Φ is not reliable—say, for example, that the exercise of Φ frequently leads to apparently paradoxical or inconsistent results. If such evidence is possible, it follows that

(C2*) Therefore, it is possible that S's socializers have nonexperiential evidence that Φ is not reliable.

The conjunction of (C2*) and (EP) entails that the justification of S's belief that p depends on whether or not her socializers have nonexperiential evidence that Φ is not reliable. Therefore,

(C3) S's justification for the belief that p is tradition-dependent.

The moral is clear. A priori knowledge is tradition-independent only if a priori justification is indefeasible. This leads to a second, more fundamental, criticism of Kitcher's argument.

In order to show that (WC) entails that a priori knowledge is tradition-dependent, Kitcher embeds (WC) within a background theory of knowledge, socio-historicism, which he acknowledges represents a departure from the epistemological tradition that includes Frege. If we make the plausible assumption that Frege's background theory of knowledge is Cartesian foundationalism, then it follows that for Frege:

(F) If S's belief that p is justified, then S's justification for the belief that p is indefeasible.

The conjunction of (F) and (A1) entails

(C) Therefore, the justification conferred on S's belief that p by $\Phi$ is not defeasible,

and Kitcher's argument for the tradition-dependence of a priori knowledge is immediately blocked. So Kitcher is faced with a dilemma. Either Frege's background theory of knowledge entails (F) or it does not. If it does not, then neither (SC) nor (WC) preserves the tradition-independence of a priori knowledge. If it does, then both (SC) and (WC) preserve the tradition-independence of a priori knowledge.

We can now generalize and sharpen the basic criticism of (SC) introduced in section I. Either (SC) is embedded in a traditional theory of knowledge that entails (F) or it is not. Within a traditional theory of knowledge, condition (b) is both unnecessary and misleading. It is unnecessary, because the possibility of experiential defeaters for beliefs justified a priori is ruled out by the general theory of knowledge. It is misleading because indefeasibility by experience is not a feature that differentiates a priori from empirical justification; it is a feature is common to both. Condition (b) is necessary only if (SC) is embedded in a nontraditional theory of knowledge that does not entail (F). But if a general theory of knowledge does not entail (F) and condition (b) differentiates a priori from empirical justification, then empirical justification is compatible with defeasibility by experience. Therefore, condition (b) imposes a higher standard on a priori justification than the general theory of knowledge requires. In the absence of some compelling supporting argument, the higher standard is *ad hoc*. Therefore, (SC) is either unnecessary or *ad hoc*.

Socio-historicism has a second important consequence. It reveals two ways in which contemporary mathematical knowledge depends on the experiences of our ancestors. First, there are "those scattered perceptions that began the whole show," and, second, there is "the division of labour [between mathematics and science] and the long sequence of experiences that have warranted our ancestors, and now us, in making that division." (Kitcher 2000, 84) Once we see this, "we'll recognize that the issue isn't one of apriorism versus empiricism, but of apriorism versus historicism, and here the interesting question is whether one can find, for logic, mathematics, or anything else, some tradition-independent warrant, something that will meet the requirements that Descartes and Frege hoped to satisfy—in short, something that will answer to the Strong conception." (Kitcher 2000, 85)

Let us provisionally grant Kitcher's claim that our mathematical knowledge depends on the experiences of our ancestors and examine its alleged consequences. The first is rooted in conceptual confusion. The important epistemological question regarding mathematical knowledge cannot be framed as a choice between historicism and apriorism because historicism is a thesis about the nature of knowledge in general; apriorism is not. The central claim of historicism is that the justification of a person's beliefs sometimes depends on the cognitive states and processes of that person's intellectual ancestors. Kitcher (2000, 81-82) contrasts the socio-historical conception of justification with "synchronic" conceptions, which hold that the justification of a person's beliefs depends only on that person's cognitive states and processes. Hence, the debate between socio-historical theories and synchronic theories is a debate over the *general* requirements for justification. The central claim of apriorism is that the beliefs that meet the general requirements on justification can be divided into two categories based on the *difference* that experience plays in meeting those requirements. Hence, apriorism is not a thesis about the general requirements for justification.

Kitcher's contention that the interesting question about mathematics is whether one can find for it some tradition-independent justification cannot be right for similar reasons. The tradition-independence of justification is a consequence of the synchronic conception of justification. Hence, the debate over whether justification is tradition-independent is a debate about the *general* requirements for justification. Consequently, when Frege claims that the justification of mathematical propositions differs from the justification of scientific propositions—that the former is a priori and the latter is not—his claim cannot be that the justification of mathematical propositions is tradition-independent but the justification of scientific propositions is not. His commitment to Cartesian foundationalism ensures that the justification of both is tradition-independent. Frege's claim is that although the justification of both is tradition-independent, there is an important difference between the role that experience plays in the justification of each.

Perhaps the source of Kitcher's confusion is the belief that the Strong conception of the a priori is essentially tied to the synchronic theory of justification. If it is, then embracing (SC) entails rejecting historicism. But it is not. Both (SC) and (WC) can be formulated within both socio-historical and synchronic theories of justification. Within a synchronic theory of justification, (WC) holds that S's belief that p is justified a priori iff the cognitive states and processes of S that justify the belief that p are exclusively nonexperiential. (SC) adds a further condition: S's nonexperiential justification is not defeasible by S's experiences. Within a socio-historical theory, (WC) holds that S's belief that p is justified a priori iff the cognitive states and processes of S *and S's intellectual ancestors* that justify the belief that p are exclusively nonexperiential. (SC) adds the further condition: S's nonexperiential justification is not defeasible by S's experiences *or those of S's intellectual ancestors*. Hence, whichever conception of justification one adopts, the two traditional questions about the a priori can be posed: What is the correct analysis of the concept of a priori knowledge? Is mathematical knowledge a priori?

IV

Let us step back and ask: What conclusions can we draw about the current debate over the nature of mathematical knowledge? The traditional debate centers around the question: Is mathematical knowledge a priori? Kitcher (1983) addresses this question directly: he offers a negative response supported by an argument whose linchpin is (SC). He (2000, 85) now rejects the importance of that question because he is pessimistic about providing an analysis of the concept of a priori knowledge:

> It seems to me that the discussions of the past decades have made clear how intricate and complex the classical notion of the a priori is, and that *neither* the Strong conception *nor* the Weak conception (nor anything else) can provide a coherent explication.

The first conclusion that we can draw is that Kitcher's pessimism is unwarranted.

The difficulty of providing a clear and coherent account of the classical conception of the a priori is due largely to a failure to distinguish the conditions constitutive of the concept of a priori knowledge from those constitutive of the more general concept of knowledge. This point emerges in two different ways in Kitcher's discussion. First, the weaknesses that he attributes to (WC) are consequences of the reliabilist theory of knowledge in which he embedded his original discussion of the a priori. Second, the features of the classical conception of the a priori that Kitcher alleges are captured only by (SC) are features of the Cartesian foundationalist theory of knowledge in which it is embedded. Once we carefully distinguish the requirements of the background theories of knowledge that Kitcher presupposes from those of the a priori, we see that he has neither offered any cogent criticisms of (WC) nor shown that (WC) is at odds with the classical conception of the a priori. Hence, his contention that (WC) fails to provide an accurate and coherent analysis of the classical conception of a priori knowledge is baseless.

Kitcher's (2000, 85) pessimism regarding the concept of a priori knowledge leads him to conclude that we should move beyond the traditional debate regarding mathematical knowledge:

> The important point is to understand the tradition-dependence of our mathematical knowledge and the complex mix of theoretical reasoning and empirical evidence that has figured in the historical process on which current mathematical knowledge is based. (Kitcher 2000, 85)

The second conclusion that we can draw is that Kitcher's new account of mathematical knowledge provides no reason to move beyond the traditional debate.

Kitcher's new account of mathematical knowledge can be formulated as follows:

(1) Socio-historicism is the correct general theory of knowledge.

(2) The experiences of our ancestors play a role in the justification of our mathematical beliefs.

(3) Therefore, our mathematical knowledge is based on a mix of theoretical reasoning and empirical evidence.

Once we recognize that the concept of a priori knowledge can be articulated within a socio-historical theory of knowledge, we also see that Kitcher's new account, when conjoined with (WC), offers a direct answer to the traditional question regarding mathematical knowledge:

(4) (WC) is the correct analysis of a priori justification.

(5) Therefore, our mathematical knowledge is not a priori.

So why not endorse (WC) and claim victory?

The third conclusion that we can draw is that Kitcher's claims about the role that experience plays in justifying the mathematical beliefs of our ancestors raises epistemological questions identical to those raised in the traditional debate regarding the nature of mathematical knowledge. His first claim is that the elementary mathematical knowledge of our early ancestors is justified by ordinary sense perception. This claim applies a familiar Millian account to the mathematical knowledge of our early ancestors. But the very same questions that have been raised regarding whether Mill has shown that *our* mathematical knowledge is empirical can be raised with respect to Kitcher's claims about the mathematical knowledge of *our ancestors*. For example, suppose that Mill provides a coherent empiricist account of the mathematical knowledge of our ancestors. It does not follow that their mathematical knowledge is not a priori unless Mill can rule out the possibility of epistemic overdetermination—i.e., the possibility that their mathematical beliefs are justified both experientially and nonexperientially.[6]

Kitcher's second claim is that the institutionalization of a division of labor in the early development of modern science in which some members were given the task of developing new mathematical concepts and principles plays a role in the justification of the mathematical beliefs of our ancestors. Evaluating this claim is more challenging since he is not fully explicit about how this epistemic division of labor shows that mathematical beliefs are justified by experience. His (2000, 84) clearest articulation comes in the following passage:

> We have learned, *from experience*, that having a group of people who think and scribble, who proceed to extend and articulate mathematical languages in the ways that mathematicians find fruitful and who provide resources for empirical science is a good thing, that creating this role promotes our inquiry.

Kitcher maintains that we have learned from experience that a division of labor promotes fruitful inquiry. But the fact that experience shows that a division of labor promotes fruitful mathematical inquiry does not entail that experience plays any role in the justification of mathematical beliefs. Perhaps Kitcher is here stressing the role of mathematics in promoting scientific inquiry and suggesting that this role is essential to the justification of mathematical beliefs. This reading of Kitcher introduces a familiar Quinean theme to the effect that the applications of mathematical theories in empirical science play an essential role in their justification. Once again, the same questions that have been raised regarding whether Quine has shown that *our* mathematical knowledge is empirical can be raised with respect to Kitcher's claims about the mathematical knowledge of *our ancestors*. For example, does the Quinean picture provide an accurate representation of our actual mathematical practices or is yet another philosophical "rational reconstruction" of a body of human knowledge of the sort that Quine and Kitcher explicitly reject?[7]

So, in the end, Kitcher's circuit through socio-historical theories of knowledge returns the current debate about mathematical knowledge to familiar territory. (WC) provides a coherent articulation of the concept of the a priori that is consonant with both the classical conception of knowledge and Kitcher's socio-historical conception. The socio-historical account of mathematical knowledge in conjunction with (WC) entails that mathematical knowledge is not a priori. The soundness of his argument rests on two familiar issues regarding mathematical knowledge. Are the experiences that are typically involved in the genesis of our mathematical beliefs (and those of our ancestors) essential to their justification? Are the empirical applications of mathematical propositions essential to their justification?[8]

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# NOTES

1. By 'general theory of knowledge or justification', I mean any theory that offers an account of the conditions necessary or sufficient for knowledge or justification.

2. Pust (2002) makes the latter point.

3. According to Kitcher (2000, 73), Parsons (1986) makes a similar point. I have not been able to locate the quotation that Kitcher attributes to Parsons.

4. Pust (2002) rejects this principle.

5. There is an alternative argument available to Kitcher. Assume that

   (A1*) S's belief that p is justified by some nonexperiential process Φ and the justification that Φ confers on S's belief that p is defeasible by experience.

   The conjunction of (A1*) and (WC) entails

   (C1*) Therefore, S's belief that p is justified a priori by Φ and the justification conferred on S's belief that p by Φ is defeasible by experience.

   If the argument from (C1) to (C3) is sound, then the argument from (C1*) to (C3) is also sound. The latter shows that (WC) leaves open (rather than entails) that a priori knowledge is tradition-dependent. I argue in the subsequent paragraph that (SC) also leaves open that possibility.

6. See Casullo (2005).

7. Maddy (1997, 184) maintains that the Quinean picture of mathematics is incompatible with his epistemological naturalism.

8. Thanks to the participants at the first annual Midwest Epistemology Workshop for their comments on an earlier version of this paper.