

# GEOMETRY OF CURVES AND SURFACES

PAUL-HERMANN ZIESCHANG

Curves and surfaces play a fundamental role in our life. They occur in engineering (car constructions, highway and roller coaster design, the development of the curve ball in baseball), in computer science (standard graphic programs such as Adobe Illustrator, construction of computer fonts such as PostScript fonts, computer-aided design), in physics (paths of particles run on curves), in optics (eye surgery), and in biology (investigation of DNA strands).

This course is a friendly and smooth introduction to the geometry of curves and surfaces. It is accessible for students who know Calculus II. We shall start with selected topics which you may be familiar with, topics such as partial derivatives of functions from  $\mathbb{R}^3$  to  $\mathbb{R}$ , derivatives of functions from  $\mathbb{R}$  to  $\mathbb{R}^3$ , and plane curves with parametric equations.

After this brief warming up we start to develop a simple and easy theory of curves in which we bring in our experiences from the above topics. The individual sections of this more systematic approach include

1. examples: straight lines, circles, helices,
2. dot product, cross product, and triple scalar product,
3. curves, speed, and acceleration on curves, vector fields on curves,
4. curvature vector field and curvature function of a curve,
5. the Frenet Formulas and the Cartan Formulas.

After this first glimpse at curves, we are prepared to look at surfaces. We start with the definition and a characterization of surfaces in  $\mathbb{R}^3$ . The individual sections of the subsequent more systematic approach to surfaces include

1. examples: cones, cylinders, saddles,
2. tangent vectors,
3. forms and the geometry of surfaces,
4. mapping of surfaces,
5. topological properties of surfaces.

The mathematical concept which is behind the notion of a surface is the one of a manifold. We shall finish our course with a smooth introduction to manifolds.