Hankel Matrices and the Vector Space of all Polynomials

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In this research project, we will study the duality between Hankel matrices of real numbers and linear functionals on the space of polynomials, as follows:
(i) Given a sequence $\beta \equiv \{\beta_n\}_{n=0}^{\infty}$ of real numbers, the associated Hankel matrix is $H_\beta := (\beta_{i+j})_{i,j=0}^{\infty}$;
(ii) Given a finite set $K \equiv \{(s_1, \rho_1), \cdots, (s_k, \rho_k)\} \subseteq \mathbb{R} \times \mathbb{R}_+$, the *generalized evaluation* on $\mathbb{R}[x]$ associated with K is the linear transformation $L_K : \mathbb{R}[x] \longrightarrow \mathbb{R}$ given by $L(p) := \sum_{i=1}^{k} \rho_i p(s_i)$ $(p \in \mathbb{R}[x])$.

We will explore the structure of Hankel matrices, esp. with respect to the canonical inner product $\langle \xi, \eta \rangle := \sum_{i=1}^{\infty} \xi_i \eta_i$, and how it relates to generalized evaluations. We will seek to associate to every positive semi-definite Hankel matrix a generalized evaluation L_K in such a way that $\beta_n = L_K(x^n)$ $(n \geq 0)$. That is, we will aim to prove that, under certain conditions, the entries of a Hankel matrix can be succinctly described by a finite set K, through its associated generalized evaluation L_K.