Analysis and modelling with nonlinear partial differential equations; or how I became resigned to waiting at traffic lights

Barbara Lee Keyfitz
University of Toronto

Friday, July 25
2:30–3:20

The talk will develop some theory of first-order partial differential equations, and will show how a simple equation is used to describe traffic flow and how it explains some things about traffic jams.

Radial Solutions of Polyharmonic Equations with Power Nonlinearities - Part I

Paul Schmidt
Auburn University

Friday, July 25
3:20–4:10

Much research has been devoted to superlinear second-order elliptic equations of the form $\Delta u = f(u)$, where is the Laplacian in $\mathbb{R}^n$ and $f : \mathbb{R} \to \mathbb{R}$ is p-homogeneous for some $p \in (1, \infty)$, say, $f(u) = \pm u|u|^{p-1}$ or $f(u) = \pm|u|^p$. One is interested, for example, in the existence, multiplicity, and qualitative properties of nontrivial solutions on $\mathbb{R}^n$ (entire solutions), nontrivial solutions on a bounded domain that vanish on the boundary (Dirichlet problem), or solutions on a bounded domain that "blow up" at the boundary (large or explosive solutions).

Higher-order equations of the same type, involving the polyharmonic operator Δm with $m \geq 2$, are of considerable interest, both from a purely mathematical standpoint and in view of applications. The higher- order theory is far from complete, with numerous open problems remaining even in the framework of radially symmetric solutions. We will discuss some of these problems and present our recent results towards their solution. The first part of the talk will focus on the structure of the set of all radially symmetric solutions and their classification, the second part on their qualitative and asymptotic behavior.

Radial Solutions of Polyharmonic Equations with Power Nonlinearities- Part II

Monica Lazzo
Universit'a di Bari

Friday, July 25
4:30–5:20

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PDE free boundary problems in tumor models

Bei Hu
University of Notre Dame

Saturday, July 26
9:00–9:50

We shall discuss the recent progress (joint work with Avner Friedman) on the PDE tumor models, the linear stability of the tumor, the nonlinear stability of the tumor, and the comparison of the stability between different models. These comparisons have implications on the physical problems.

Principal eigenvalue of an elliptic operator with large advection and evolution of disperseral

Yuan Lou
Ohio State University

Saturday, July 26
9:50–10:40

We investigate the asymptotic behavior of the principal eigenvalue of an elliptic operator as the coefficient of the advection term approaches infinity. As a biological application, a Lotka-Volterra reaction-diffusion-advection model for two competing species in a heterogeneous environment is studied. The two species are assumed to be identical except their dispersal strategies: both species disperse by random movement and advection along environmental gradients, but one species has stronger biased movement than the other one. It is shown that at least two scenarios can occur: if only one species has a strong tendency to move upward the environmental gradients, the two species will coexist; if both species have such strong biased movements, the species with the stronger biased movement will go to extinct. These results provide a new mechanism for the coexistence of competing species, and they also suggest that an intermediate biased movement rate may be evolutionary stable. This talk is mainly based on some joint works with Xinfu Chen and Richard Hambrock.

Trajectory Attractors for Reaction Diffusion Problems from Climate Modeling

Georg Hetzer
Auburn University

Saturday, July 26
11:00–11:50

Energy balance climate models describe the evolution of a long-term mean of temperature by employing the relevant balance equations for the heat fluxes involved. The horizontal heat flux is parameterized by a diffusion operator, and here we include a bio-feedback by introducing a Volterra map V on a suitable function space. A typical example for the resulting reaction-diffusion problem is

$$\begin{cases} c(x)\partial_t u - \nabla\cdot[k(x)\,|\nabla u|^{p-2}\,\nabla u] + g(u, V(u|_{[0,\infty)},\phi)(t)) \\ \qquad\qquad \in F(t,x,u,\overline{u},V(u|_{[0,\infty)},\phi)(t)) \qquad t>0,\ x\in M, \\ \overline{u}(t,x) := \int_{-T}^{0}\beta(s,x)u(t+s,x)\,ds,\ t>0,\ x\in M, \\ u(s,x) = u_0(s,x), \qquad\qquad -T\le s\le 0, x\in M. \end{cases}$$

One is interested in nonnegative solutions $u = u(t, x)$ (temperature in Kelvin). M is a closed, compact, oriented Riemannian surface representing the Earth's surface, the positive functions c and k represent the thermal inertia and the diffusivity of the system, respectively, F stands for the absorbed solar radiation flux, and g represents the emitted terrestrial radiation flux.

A suitable mathematical framework for establishing the existence of a global attractor will be discussed in this talk.

Divergence-Measure Fields, Sets of Finite Perimeter,and Nonlinear Conservation Laws

Gui-Qiang Chen
Northwestern University

Saturday, July 26
2:00–2:50

We will discuss a class of weakly differentiable vector fields, called divergence-measure fields, and its natural connection to entropy solutions for conservation laws. In particular, we will discuss some recent efforts to develop the theory of divergence-measure fields toward constructing analytical frameworks for studying solutions of multidimensional hyperbolic conservation laws. Further connections, trends, and open problems in this direction will be also addressed.

Almost global wellposedness of the 2-D full water wave problem

Sijue Wu
University of Michigan

Saturday, July 26
2:50–3:40

We consider the problem of global in time existence and uniqueness of solutions of the 2-D infinite depth full water wave equation. It is known that this equation has a solution for a time period $[0, T^{\epsilon}]$ for initial data of type $\epsilon\Phi$, where T depends only on Φ. We show that for such data there exists a unique solution for a time period $[0, e^{T/\epsilon}]$. This is achieved by some better understandings of the nature of the nonlinearity of the water wave equation.

Rigorous Derivation of the Hydrodynamical Equations for Rotating Superfluids

Hailiang Liu
Iowa State University

Saturday, July 26
4:00–4:50

Using a modified WKB approach, we present a rigorous semi-classical analysis for solutions of nonlinear Schrödinger equations with rotational forcing. This yields a rigorous justification for the hydrodynamical system of rotating superfluids. In particular it is shown that global-in-time semi-classical convergence holds whenever the limiting hydrodynamical system has global smooth solutions and we also discuss the semi-classical dynamics of several physical quantities describing rotating superfluids.

About the stability of rotating gas balls

Gerhard Strohmer
University of Iowa

Saturday, July 26
4:50–5:40

We consider the question of nonlinear stability of the equilibrium states of barotropic, self-gravitating viscous fluids which are slowly rotating like a rigid body. These equilibrium states as well as the non-stationary solutions occupy part of space, and a constant pressure is assumed on the free surface, but no surface tension. Although the rotation is slow, the stability of these equilibria cannot be obtained by a simple perturbation argument from the case of a non-rotating configuration, as the disturbances of the surfaces even in the non-rotating case do not necessarily decay exponentially.

The Action-Dependent Wave Function Problem: Well Posedness and Efficient Numerical Approximation of Solutions

A. J. Meir
Department of Mathematics and Statistics
Auburn University

Sunday, July 27
9:00–9:50

We describe some results for a mixed (elliptic-hyperbolic) partial differential equation and an efficient multigrid algorithm for the numerical approximation of its solutions.

The problem models an equation which arises in atomic physics and describes a new object: the "action-dependent wave function."

This is joint work with Irad Yavneh, Department of Computer Science, Technion - Israel Institute of Technology, Haifa, Israel.

A semiconductor model with temperature effect

Xiangsheng Xu
Mississippi State University

Sunday, July 27
9:50–10:40

In this talk we will present a semiconductor model which takes into account the current generated by the gradient of the temperature. The model involves quadratic nonlinearities. A new interpolation inequality is developed, and this enables us to establish an existence assertion for the model.

Wave front solutions in the theory of boiling liquids

Ruediger Landes
University of Oklahoma

Sunday, July 27
10:40–11:30

In order to model the phase transition from nucleate to transient boiling Professor Marquardt from Aachen proposed to consider the the heat equation in the wall of the heater with a nonlinear Neumann boundary condition.

For the one-dimensional heat equation with a nonlinear inhomogeneous term the existence of wavefront solutions is well known. Work of Aronson and Weinberger also dealt with the more-dimensional situation and showed that there are "sub-solutions "which behave like wave fronts.

Here we discuss an approach which provides a wavefront type sub-solution for the nonlinear Neumann problem. The issues involved are:

• The existence and comparison principle for weak solutions.
• The construction of a subsolution for the wavefront involving monotonicity and regularity properties of solutions of an elliptic nonlinear Neumann problem.
• Estimates of the speed of the the subsolution.
• Stable initial configurations.