

Saturday, June 28 (morning)

8:00–8:30 Snacks in Muhly Lounge

 8:30–9:20 **Jean-Pierre Gabardo** (MacMaster University)  
 Tiling problems in wavelet and Gabor analysis.

The main goal of this talk is to show how certain questions dealing with wavelet or Gabor type expansions lead naturally to tiling problems. In the case of wavelet analysis, we will discuss the construction of wavelet sets satisfying certain self-similarity conditions. In the case of Gabor expansions, we will explain the relationship between the existence of Gabor generators for certain subspaces and the construction of tiles associated with lattices.

 9:30–9:50 **Greg Ongie** (University of Iowa)  
 Orthogonal polynomials on the Cantor set.

An orthogonal polynomial sequence (OPS) is traditionally defined by means of Riemann integration, but more generally an OPS can be defined by means of integration with respect to a measure. Using an iterated function system, we derive a family of probability measures having the middle-thirds Cantor set as their support, and we verify the existence of an OPS associated with each of these measures by examining the positivity of their moment matrices. Then, using the Gram-Schmidt method and a recursive formula for the moments, we show how to calculate an explicit representation of the polynomials in these OPS's. Finally, we also derive various properties of these polynomials and compare our results to those from the theory of classical orthogonal polynomials.

 10:00–10:20 **Myung-Sin Song** (Southern Illinois University Edwardsville)  
 Analysis of Fractals, Image Compression and Entropy Encoding.

In this talk we show that algorithms in a diverse set of applications may be cast in the context of relations on a finite set of operators in Hilbert space. The Cuntz relations for a finite set of isometries form a prototype of these relations. Such applications as entropy encoding, analysis of correlation matrices (Karhunen-Loève), fractional Brownian motion, and fractals more generally, admit multi-scales. In signal/image processing, this may be implemented with recursive algorithms using subdivisions of frequency-bands; and in fractals with scale similarity.

10:30–11:00 Break

 11:00–11:20 **Doug Hardin** (Vanderbilt University)  
 Packing and discrete minimum energy problems on fractal sets.

We consider asymptotic properties (as  $N \rightarrow \infty$ ) of 'ground state' configurations of  $N$  particles restricted to a  $d$ -dimensional compact set  $A \subset \mathbf{R}^p$  that minimize the Riesz  $s$ -energy functional

$$\sum_{i \neq j} \frac{1}{|x_i - x_j|^s}$$

for  $s > 0$ . As  $s \rightarrow \infty$ , such configurations approach 'best-packing' configurations.

 11:30–11:50 **Matt Calef** (Vanderbilt University)  
 Riesz  $d$ -energy on sets of Hausdorff dimension  $d$ .

The Riesz  $s$ -energy  $I_s(\mu)$  of a probability measure  $\mu$  supported on a set  $A$  of Hausdorff dimension  $d$  is infinite when  $s$  is greater than or equal to  $d$ . We consider a normalized energy

$$\tilde{I}_d(\mu) = \lim_{s \rightarrow d^-} (d - s)I_s(\mu)$$

and show that it is well-defined for all probability measures supported on  $A$  and is uniquely minimized by normalized Hausdorff measure restricted to  $A$ .

11:50–2:00 Lunch

Saturday, June 28 (afternoon)

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11:50–2:00 Lunch

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2:00–2:20 **Keri Kornelson** (Grinnell College)

Moments of equilibrium measures for iterated function systems.

Every iterated function system has an associated equilibrium measure, and hence an associated moment matrix. If the IFS is affine, the moments can be exactly determined. If not, they can be approximated in many cases via finite matrix multiplication.

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2:30–2:40 **Talent Takundwa** (Grinnell College)

Attractors of Iterated Function Systems.

We will cover the basics of Iterated Function Systems (IFS) — in particular, contraction mappings, Banach's theorem, attractors, and measures. The talk will conclude with a fairly detailed analysis of the analysis of the IFS whose attractor is the Cantor Set.

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2:50–3:00 **Pengjun Shen** (Grinnell College)

Hutchinson Measure of the Overlap.

In this talk, we introduce Hutchinson measure and show how to compute the measure of overlap intervals using pattern we have discovered in our summer 2008 research.

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3:00–3:20 Break

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3:20–3:30 **Fai Tosuratana** (Grinnell College)

Measuring Overlaps of Fractals in  $\mathbb{R}$  and  $\mathbb{R}^2$ .

In this talk, we focus on fractal constructions in  $\mathbb{R}$  and  $\mathbb{R}^2$  with mappings  $\tau_i(x) = \lambda(x + u_i)$ ,  $i = 0, 1$  in  $\mathbb{R}$  and  $i = 0, 1, 2$  in  $\mathbb{R}^2$ . We examine what happens at different  $\lambda$ 's as seen in a 2007 paper by Jorgenson, Kornelson, and Shuman. This paper is the basis for research questions the speakers would like to work on this summer. The previous two talks will provide sufficient background.

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3:40–4:00 **Karen Shuman** (Grinnell College)

Bernoulli measures and Fourier transforms.

In this talk we examine a problem about invariant measures of certain Bernoulli IFSs which was solved by Paul Erdős in 1939, and we obtain Erdős-like results in higher dimensions. The background on Bernoulli IFSs provided in the previous talks will be sufficient to understand this talk. Joint work with Palle Jorgensen (University of Iowa) and Keri Kornelson (Grinnell College, University of Oklahoma).

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Sunday, June 29 (morning)

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 8:00–8:30 Snacks in Muhly Lounge
 

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 8:30–9:20 **Jun Kigami** (Kyoto University, Graduate School of Informatics)  
 Analysis on Fractals.

This is a survey on analysis on fractals from 1980's to the present. I will focus on the standard Laplacian and/or Dirichlet form on the Sierpinski gasket. Beginning from the basic definition, I will explain the recent result on the measurable Riemannian geometry on the Sierpinski gasket.

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 9:30–10:20 **Luke Rogers** (University of Connecticut)  
 Analysis on the Basilica Julia set.

Among the most natural and interesting classes of fractals are the Julia sets of complex dynamical systems. I will talk about recent work with Alexander Teplyaev in which we construct a Kigami-type analysis on one example, the Basilica Julia set, and determine some spectral information about the resulting Laplacian.

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 10:30–11:00 Break
 

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 11:00–11:20 **Huojun Ruan** (Zhejiang University and Cornell University)  
 Gap sequence, Lipschitz equivalence and box dimension of fractal sets.

We introduce a notion of *gap sequences* for compact sets  $E \subset \mathbb{R}^d$ , which is a generalization of the gap sequences of compact sets on the real line. We show that if the gap sequences of two fractal sets are not equivalent, then these two sets can not be Lipschitz equivalent, where the later fact is usually very hard to verify. Finally, we show that for some typical fractal sets, the gap sequences characterize the upper box dimension.

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 11:30–11:50 **Yang Wang** (Michigan State University)  
 Refinable Piecewise Polynomial Functions.  
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 11:50–2:00 Lunch
 

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Sunday, June 29 (afternoon)

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 11:50–2:00 Lunch
 

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 2:00–2:50 **John Rock** (CSU Stanislaus)

**Multifractal spectra and abscissa of converge functions for some multinomial measures.**

Certain aspects of a multifractal measure such as the Binomial Measure can be described with a spectrum of real dimensions called the multifractal spectrum. Results from the study of fractal strings such as the Cantor String via analysis of certain zeta functions and their complex dimensions (poles) provide the motivation for a new approach to finding these multifractal spectra. A direct connection exists between the abscissa of convergence function for a certain family of zeta functions and the multifractal spectra of a multinomial measure. Specifically, for a countable and dense subset of parameter values associated with a fixed multinomial measure, the multifractal spectrum and abscissa of convergence functions are identical.

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 3:00–3:50 **Hung Lu** (Hawai'i Pacific University)

**Self-similar  $p$ -adic fractal strings and their complex dimensions.**

First, we discuss real (or archimedean) fractal strings. Then, we discuss  $p$ -adic numbers. Finally, we talk about  $p$ -adic (or nonarchimedean) fractal strings and their complex dimensions. Specifically, the focus is on self-similar  $p$ -adic fractal strings. We obtain a closed-form formula for the geometric zeta functions and show that these zeta functions are rational functions in an appropriate variable. We also prove that every self-similar  $p$ -adic fractal string is lattice. Finally, we define the notion of self-similar set and discuss its relationship with that of self-similar  $p$ -adic fractal strings. We illustrate the general theory by two simple examples, the nonarchimedean Cantor and Fibonacci strings.

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 4:00–4:10 Break
 

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 4:10–5:00 **Erin Pearse** (University of Iowa)

**Operator theory of electrical resistance networks.**

An electrical resistance network (ERN) is a graph with edges weighted by a function whose values are interpreted as resistances; the resulting voltage drop between two vertices gives an intrinsic metric on the network. One may embed this network in a Hilbert space and use the resulting framework to understand various function spaces associated to the network. I will discuss how the presence of harmonic functions affects the relationship between the Dirichlet energy form and the (discrete) Laplace operator, and how this may be understood in terms of a certain notion of *boundary* on infinite graphs. The structure of the Hilbert space allows for easy solutions to some potential-theoretic problems and reveals the asymptotic geometry of the network.

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